WHO WAS PASCAL AND WHAT'S THIS ABOUT HIS TRIANGLE?

Subject Matter: Mathematics
Grade Levels: 6-8
Time Allotment: 2 hours
Master Teacher: Karen Mapes

Overview

Students use the patterns in Pascal's Triangle to build number sense and do some algebraic thinking.

Learning Objectives

Students will be able to:

- Recognize powers of 2, prime numbers and triangular numbers.
- Describe patterns in terms of numbers and operations.
- Investigate and describe geometric and exponential patterns.
- Identify, describe and generalize patterns.

Oregon Standards Available at:

http://www.ode.state.or.us/cifs

Mathematics - Calculations and Estimations

- Perform calculations on whole numbers, fractions, decimals and integers using paper and pencil, calculators and/or computers.

Mathematics - Statistics and Probability

- Design and carry out simulations to compare experimental and theoretical probability and to make predictions.
- Plan and conduct experiments and simulations using data to make predictions or support arguments.

Mathematics - Algebraic Relationships

- Recognize, create, describe and analyze patterns and sequences (arithmetic and geometric).

Mathematical Problem Solving

- Use pictures, models, diagrams and symbols to show main mathematical concepts in the problem.

National Standards From the National Council of Teachers of Mathematics

(http://standards.nctm.org/)

In grades 5-8, the mathematics curriculum should include explorations of patterns and functions so that students can:

- Describe, extend, analyze and create a wide variety of patterns.
- Describe and represent relationships with tables, graphs and rules.
- Analyze functional relationships to explain how a change in one quantity results in a change in another.
- Use patterns and functions to represent and solve problems.

Media Components

Web

Math Forum Web Sites
The Math Forum is a Swarthmore Web site with many extremely useful educational lessons. You will be using quite a few of their Pascal's Triangle resources.

- This page includes the intermediate size of Pascal's Triangle without the numbers filled in.
  http://mathforum.org/workshops/usi/pascal/images/midd.comb1.gif

- This page includes the intermediate size of Pascal's Triangle with the numbers filled in.
  http://mathforum.org/workshops/usi/pascal/images/midd.comb2.gif

- This page includes an example of how the artwork in the Culminating Activity might look.
  http://mathforum.org/workshops/usi/pascal/mid.color_pascal.html

Tripod Web Site

- Most of the Learning Activity patterns are found on this Web site: http://ptri1.tripod.com/

Pascal Biography Web Sites
Sites with biographical material on Pascal are listed below:

- http://www.island-of-freedom.com/PASCAL.HTM
- http://www.maths.tcd.ie/pub/HistMath/People/Pascal/RouseBall/RB_Pascal.html
- http://www.kirjasto.sci.fi/bpascal.htm
- http://www.wsu.edu/~dee/ENLIGHT/PASCAL.HTM

Materials

- A computer that is connected to a projector or television to display patterns on the Web sites
- A blank copy of Pascal's Triangle for each student for the Introductory Activity
- You may want a number-filled copy of Pascal's Triangle for less capable students to use during the Learning Activities

For Culminating Activity:

- A number-filled copy of Pascal's Triangle for each student
- Coloring tools in 6 colors
- Scissors
- Construction paper
- Paste or glue
- A set of Coloring Rules for each group (at end of lesson plan)
- Transparency of the Color-Number Key (at end of lesson plan)

Prep for Teachers

Photocopy blank Pascal's Triangles, number-filled Pascal's Triangles and the coloring rules.

Bookmark the Web sites you will be using with the projector.

Make a "key" copy of what each of the coloring-rule triangles should look like.

Make a transparency of the Color-Number Key.

Familiarize yourself with Pascal's biography for the Introductory Activity.

Introductory Activity

Step 1: What Can You Do With a Triangle?

Before you hand out the worksheet, provide students with a **Focus for Media Interaction** by asking them to think about what the paper they are about to receive might have to do with patterns.

Give students a copy of the worksheet found at the Math Forum site at http://mathforum.org/workshops/usi/pascal/images/midd.comb1.gif.

This copy of Pascal's Triangle can be blown up on a copy machine to 125% of its original size to give students more room to work.

Ask students for feedback on which pattern might fit into this triangular shape. List answers on an overhead, chalkboard or chart.

Step 2: Introducing Pascal

Explain that the triangular pattern they have in front of them dates back to the 1600s in France. It was created by the French mathematician Blaise Pascal who began with a triangle by having each number formed from the sum of the two above it.

Pascal did a lot of other things: he was a Christian philosopher, a mathematician, a scientific thinker and a physicist. He invented one of the first digital calculators (not run on electricity, of course) to help his father, who was a tax collector. Web sites with biographies of Pascal are listed in the Media Components: Web section of this lesson plan; use the sites you feel are appropriate for your students.

Step 3: Creating the Triangle

Give students the working rule for their pattern: put the number 1 at the top. The number 1 also goes at the start and end of each line. The middle numbers that fill each middle hexagon are determined by the sum of the two numbers in the hexagons above it.

Students can refer to this Web site to see how they should be completing the Pascal's Triangle Pattern: http://mathforum.org/workshops/usi/pascal/images/midd.comb2.gif

Learning Activities

Once students have a copy of Pascal's Triangle, they can begin to learn the significance of the patterns it contains.

Have students work in pairs or groups to analyze the following patterns found in Pascal's triangles. The following patterns were taken from http://ptri1.tripod.com/, but can also be found

other places on the Web. As you discuss each pattern, show students the material on the Web site.

Step 1: Provide students with a **Focus for Media Interaction** by asking: "What do you notice about the sums of the rows and the powers of two?"
Answer: The sum of the numbers in any row is equal to 2 to the nth power or 2n, when n is the number of the row. For example:

$2^0 = 1$
$2^1 = 1+1 = 2$
$2^2 = 1+2+1 = 4$
$2^3 = 1+3+3+1 = 8$
$2^4 = 1+4+6+4+1 = 16$

Step 2: Provide students with a **Focus for Media Interaction** by asking: "What do you notice about the rows that start, after the 1, with prime numbers?"

Answer: If the first element in a row is a prime number, all the numbers in that row (excluding the 1's) are divisible by it. For example, in row 7 (1 7 21 35 35 21 7 1) 7, 21 and 35 are all divisible by 7.

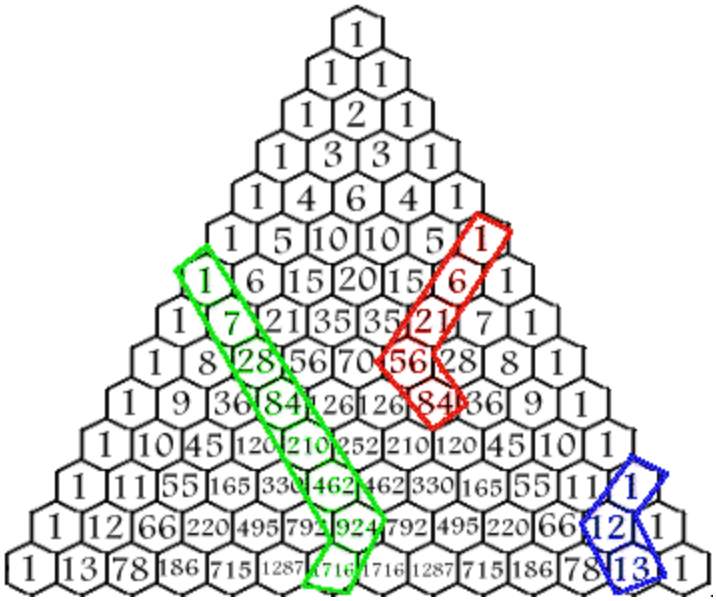
Step 3: Provide students with a **Focus for Media Interaction** by asking: "What do you notice about the number formed by each row and the powers of 11?"

Answer: If a row is made into a single number by using each element as a digit of the number (carrying over when an element itself has more than one digit), the number is equal to 11 to the nth power or 11n when n is the number of the row the multi-digit number was taken from.

Row #	Formula	=	Multi-Digit number	Actual Row
Row 0	11^0	=	1	1
Row 1	11^1	=	11	1 1
Row 2	11^2	=	121	1 2 1
Row 3	11^3	=	1331	1 3 3 1
Row 4	11^4	=	14641	1 4 6 4 1
Row 5	11^5	=	161051	1 5 10 10 5 1
Row 6	11^6	=	1771561	1 6 15 20 15 6 1
Row 7	11^7	=	19487171	1 7 21 35 35 21 7 1
Row 8	11^8	=	214358881	1 8 28 56 70 56 28 8 1

Step 4: Provide students with a **Focus for Media Interaction** by asking: "What do you notice about the hockey stick pattern?" (You will need to demonstrate what that means with an overhead of the triangle. Hockey stick patterns are shown below.

Answer: If a diagonal of numbers of any length is selected starting at any of the 1's bordering the sides of the triangle and ending on any number inside the triangle on that diagonal, the sum of the numbers inside the selection is equal to the number below the end of the selection that is not on the same diagonal itself. If that seems unclear, review the drawing.
1+6+21+56 = 84
1+7+28+84+210+462+924 = 1716
1+12 = 13



Step 5: Provide students with a **Focus for Media Interaction** by asking: "Where do you find triangular numbers in the triangle?"

Answer: The triangular numbers can be found in the diagonal starting at row 3 as shown in the diagram. The first triangular number is 1, the second is 3, the third is 6, the fourth is 10 and so on.

Culminating Activity

In this activity, students reinforce their knowledge about the patterns in Pascal's Triangle by coloring patterns of multiples. Triangles that are similarly colored are then put together to form hexagons and posted as artwork in the classroom.

Form students into six groups. There will probably be four or five students in a group. If you have more students in a group, have a less capable student double up with another. There are five coloring-rule sequences.

Each student will need a copy of Pascal's Triangle with the numbers printed on it. Each student in the group will have a different rule about multiples by which to color his or her triangle. Students will all need to use the same colors and coloring tools so that the triangles will look similar when they are put together. At the end of the lesson plan, you will find a Color-Number Key you can make that will assign specific colors to numbers. Students should all be using the same coloring media (colored pencils, felt pens or crayons).

Students are given a strip of paper that explains how to color their triangles. The Coloring Rules below are also included at the end of the lesson plan.

Rule 1: Color all of the odd numbers color #1 and the even numbers color #2. These are the multiples of 2 and the numbers that are not multiples of 2.

Rule 2: Color the multiples of 3 with color #3. Color the multiples of 2 that have not already been colored in the "multiple of 3" color with color #2. Color all the rest of the numbers (that are not multiples of 2 or 3) in color #1.

Rule 3: Color the multiples of 4 with color #4. Color the multiples of 3 that have not already been colored in the "multiple of 4" color with color #3. Color the multiples of 2 that have not already been colored with color #2. Color all the rest of the numbers (that are not multiples of 2, 3 or 4) in color #1.

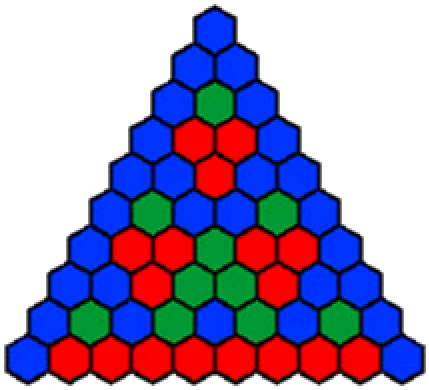
Rule 4: Color the multiples of 5 with color #5. Color the multiples of 4 that have not already been colored in the "multiple of 5" color with color #4. Color the uncolored multiples of 3 with color #3 and then the uncolored multiples of 2 with color #2. Color all the rest of the numbers (that are not multiples of 2, 3, 4 or 5) in color #1.

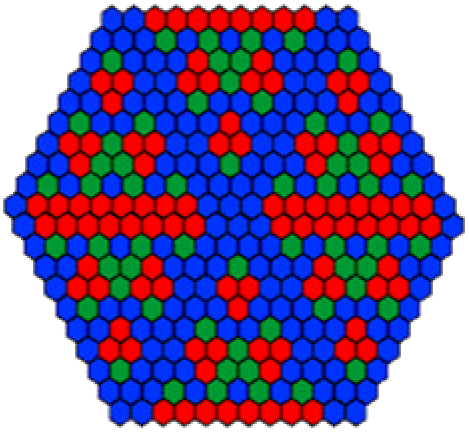
Rule 5: Color the multiples of 6 with color #6. Color the multiples of 5 that have not already been colored in the "multiple of 6" color with color #5. Color the uncolored multiples of 4 with color #4, then the uncolored multiples of 3 with color #3 and then the uncolored multiples of 2 with color #2. Color all the rest of the numbers (that are not multiples of 2, 3, 4, 5 or 6) in color #1.

Re-form your groups so that the students who colored according to the same rule are all together in a group. Have them compare their patterns. If the patterns are not the same, tell them to figure out why. They can cut out their triangles and paste them together on colorful construction paper as a piece of artwork that will illustrate their rule.

An example of how the artwork might look can be found at the Math Forum Web site (Note: The author of that lesson uses different coloring rules):
http://mathforum.org/workshops/usi/pascal/mid.color_pascal.html

Six identically colored triangles can be joined to form a hexagon. These constructions make great classroom or hall decorations. Looking at the center point gives the optical illusion of a cube in three dimensions.





Cross-Curricular Extensions

Social Studies

- Learn about the world in which Pascal lived.

Science

- Learn about Descartes, a contemporary of Pascal's, and the scientific method.
- Learn about Pascal's computer and other early computers, such as those of Charles Babbage and Ada Lovelace.

Math

- Learn about Sierpinski's Triangle and fractals. There is some information about this on the Tripod site.
- Extend the lesson to talk about Fibonacci numbers on Pascal's Triangle.

Art

- There is information on the Math Forum Web site about how to connect several copies of Pascal's Triangle (from the Culminating Activity) into a star.

Language Arts

- Make a poster out of the Culminating Activity artwork with a written explanation of the pattern that was used.

Community Connections

- Donate artwork from the Culminating Activity to a senior center.

Coloring Rules

These are the coloring rules that can be cut out in strips:

Rule 1:

- Color all of the odd numbers color #1 and the even numbers color #2.
- These are the multiples of 2 (color #2) and the numbers that are not multiples of 2 (color #1).

Rule 2:

- Color the multiples of 3 with color #3.
- Color the multiples of 2 that have not already been colored in the "multiple of 3" color with color #2.
- Color all the rest of the numbers (that are not multiples of 2 or 3) in color #1.

Rule 3:

- Color the multiples of 4 with color #4.
- Color the multiples of 3 that have not already been colored in the "multiple of 4" color with color #3.
- Color the multiples of 2 that have not already been colored with color #2.
- Color all the rest of the numbers (that are not multiples of 2, 3 or 4) in color #1.

Rule 4:

- Color the multiples of 5 with color #5.
- Color the multiples of 4 that have not already been colored in the "multiple of 5" color with color #4.
- Color the uncolored multiples of 3 with color #3 and then the uncolored multiples of 2 with color #2.
- Color all the rest of the numbers (that are not multiples of 2, 3, 4 or 5) in color #1.

Rule 5:

- Color the multiples of 6 with color #6.
- Color the multiples of 5 that have not already been colored in the "multiple of 6" color with color #5.
- Color the uncolored multiples of 4 with color #4, then the uncolored multiples of 3 with color #3 and then the uncolored multiples of 2 with color #2.
- Color all the rest of the numbers (that are not multiples of 2, 3, 4, 5 or 6) in color #1.

Color-Number Key

Color #1 _____

Color #2 _____

Color #3 _____

Color #4 _____

Color #5 _____

Color #6 _____