THE PATTERNS OF FIBONACCI

Subject Matter: Mathematics – Algebraic Understanding, Patterns
Grade Levels: 7-8
Time Allotment: 1-2 hours
Master Teacher: Karen Mapes

Overview

The Fibonacci series is a very dynamic application of mathematical patterns in the real world. Fibonacci numbers can be found in many aspects of nature – the number of petals on a flower are almost always a Fibonacci number, the number of scales that make a spiral on a pinecone or pineapple are Fibonacci numbers, and the proportions of rectangles that create a spiral in a snail shell are Fibonacci numbers.

Fibonacci was a 13th-century Italian mathematician. He is credited with introducing the decimal number system to Europe and with discovering a pattern that seems to occur over and over in nature in various ways. The pattern was named after him – the Fibonacci series. The series itself is fairly simple, but its applications in math and nature can become quite complicated.

The Fibonacci Series:

The series begins with 0 and 1. After that, use the simple rule:

> **Add the last two numbers to get the next.**
>
> 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987 ...

You might ask where this came from? In Fibonacci's day, mathematical competitions and challenges were common. In 1225, Fibonacci took part in a tournament at Pisa ordered by the emperor himself, Frederick II. It was in just this type of competition that the following problem arose:

> Beginning with a single pair of rabbits, if every month each productive pair bears a new pair, which becomes productive when they are 1 month old, how many rabbits will there be after n months? (http://plus.maths.org/issue3/fibonacci/)

Learning Objectives

Fibonacci numbers can be quite complicated and branch into many areas of nature, art and mathematics. This lesson focuses on having students recognize the origin of the Fibonacci series and its relationship to the spiral shape.

Students will be able to:

- Analyze the number patterns found within the Fibonacci series.
- Comprehend the application of the Fibonacci series in several real-world situations.

Oregon Standards available at:

http://www.ode.state.or.us/cifs

Mathematics - Algebraic Relationships

- Represent relationships among variables using words, tables, graphs, patterns, generalizations and equations.
- Analyze relationships to explain how a change in one quantity results in a change in another.
- Recognize, create, describe and analyze patterns and sequences (arithmetic and geometric).

National Standards From the National Council of Teachers of Mathematics

(http://www.nctm.org)

In grades 5-8, the mathematics curriculum should include explorations of patterns and functions so that students can:

- Describe, extend, analyze and create a wide variety of patterns.
- Describe and represent relationships with tables, graphs and rules.
- Use patterns and functions to represent and solve problems.

In grades 5-8, the mathematics curriculum should include explorations of algebraic concepts and processes so that students can:

- Represent situations and number patterns with tables, graphs, verbal rules and equations and explore the interrelationships of these representations.
- Analyze tables and graphs to identify properties and relationships.
- Apply algebraic methods to solve a variety of real-world and mathematical problems.

Media Components

Video

Check the link at http://www.opb.org/edmedia/trs/ to find access to the video(s) from unitedstreaming™ referenced in this lesson plan.

- "Mathematical Eye: Fibonacci and Prime Numbers" (20:16)
  - **Clip:** "Investigating Numbers in the Natural World" (03:36)
  - **Clip:** "Investigating Fibonacci Numbers" (05:45)

Web

- **Ron Knott's Surrey University Web Site**
  This Web site includes information on the Fibonacci numbers, the Golden section and the Golden string. Good site for all things Fibonacci; good illustrations.
  http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html - Rabbits

- **R. Knott, D. A. Quinney and PASS Maths**
  This site sums up Fibonacci's contributions, and includes a picture and biographical information.
  http://plus.maths.org/issue3/fibonacci/

Materials

- Several sets of an assortment of flowers, twigs with leaves, pinecones and bunches of pine needles that show Fibonacci numbers occurring in nature
- Optional: a cauliflower, an African violet plant and a pineapple. If you can get it, bring in a chambered Nautilus seashell that has been cut in half.
- Copies of the Fibonacci and the Rabbits Worksheet (at end of lesson plan)
- Computer and projector

Prep for Teachers

When using media, provide students with a **Focus for Media Interaction**, a specific task to complete and/or information to identify during or after viewing of video segments, Web sites or other multimedia elements.

Download the video clips that you will be using. You can preview them by streaming them on the computer, but when showing them in the classroom, you'll want to use a downloaded copy – either on the computer hard drive or burned onto a CD – because you have more control that way. Make sure you have a copy of the free Windows Media Player to use for watching video clips. For many clips, it is also possible to use QuickTime Player.

Be familiar with the parts of the video that will be shown and the cues within the videos at which you will **start** and **pause**. **Cue** the first video clip, "Investigating Numbers in the Natural World" (03:36), from the video, "Mathematical Eye: Fibonacci and Prime Numbers" (20:16), to the section right after the title by using the scroll bar, then **pause** it at approximately 00:10.

Explore the two Web sites to be used. Examine the links and decide how much time and detail you want. Bookmark the sites so that you can get to them quickly during the lesson.
Divide students into enough groups so that each has a set of natural materials.

Introductory Activity

Students will notice the Fibonacci series through the following series of video clips and activities:

Step 1: Without telling students anything at all about the Fibonacci series of numbers, show them the following video clip. Before starting the clip, provide your students with a **Focus for Media Interaction** by asking them, "What do you think the kids in the video are doing?" **Play** the video clip, "Investigating Numbers in the Natural World" (03:36), from the video, "Mathematical Eye: Fibonacci and Prime Numbers" (20:16), from the point where you cued the video as the camera is panning up to the bridge and showing the students on the bridge counting something. (Remember, you are skipping the first part with the title, since it mentions Fibonacci and some students may have already heard of the series.) **Pause** the video at approximately 00:28 when the boy writes something on his paper. (Ask students what they think is going on. You might have them tell a neighbor first and see if the class can come to a consensus. "Counting something" is probably obvious, but try to get them to predict what.)

Provide your students with a **Focus for Media Interaction** by asking them to watch for a chance to count the scales on a turtle's back. **Play** the video as it shows the turtle until about 00:37. **Pause** the video to give students a chance to count the scales (there should be 13), then **play**. **Pause** after the narrator says, "… Do you think it crops up anywhere else?" (00:46) Ask the students what they think. If 13 seems like a strange number, what would be a less strange natural number? Where else might they expect to find 13?

Step 2: Tell students they are going to investigate what numbers do appear in nature and keep track. They should work in groups with one student recording the results. Give each group a set of natural materials. Include flowers, sets of pine needles, twigs with leaves still on them and pinecones. (Prepare ahead of time to have objects with Fibonacci numbers of things, not the exceptions to the rule.) Have students count and write down the numbers they find represented on the object and list in what ways they are represented. They should be coming up with things like number of petals, number of leaves on a twig, number of scales in a pinecone and number of pistils and stamen on a flower.

After the list is complete, compile all group lists into a master list put into numerical order. Discuss any patterns they notice. Students may or may not at this point be able to spot similar

numbers. Ask them, "Did the number 13 come up again? Did it come up in your flower petal counts? What do you think is going on? Is there a pattern you can describe?" Take down ideas.

Tell students, "Let's look at another way to view this pattern – one that has been around since the 13th century …"

Learning Activities

Step 1: Introducing Fibonacci
Provide your students with a **Focus for Media Interaction** by asking them, "What did the Europeans use to count before they had the digits 0-9? (Roman numerals) Go to your bookmark for the Web site, http://plus.maths.org/issue3/fibonacci/ to show a picture of Fibonacci and a brief biography. Show on your screen the section, "Have you ever wondered where we got our decimal numbering system from?" through the quote from Fibonacci's book. Allow students time to discover the answer to your question and then discuss it.

Step 2: Introducing the Fibonacci Sequence
Provide your students with a **Focus for Media Interaction** by asking them to look for a pattern in the series of numbers for which Fibonacci is famous. Scroll past the "Root Finding" section to the "Fibonacci Sequence" section. Ask students to study the numbers in the Fibonacci series and see if they can spot a pattern. Depending on your class, you can either leave the answer showing ("Add the last two numbers to get the next.") or you can arrange the screen so that statement is just past the top where it can't be seen.

Provide your students with a **Focus for Media Interaction** by asking them, "What do you think this rabbit problem has to do with this sequence of numbers?" Direct their attention to the rabbit problem with which Fibonacci was challenged.

Step 3: Solving the Rabbit Problem
Provide your students with a **Focus for Media Interaction** by asking them to watch for some of the same biographical information they just saw on the Web site. Use the beginning of the video clip, "Investigating Fibonacci Numbers" (05:45), from the video, "Mathematical Eye: Fibonacci and Prime Numbers" (20:16), to support this part of the lesson.

Play from the beginning of the tape. **Pause** after the narrator says, "… How many rabbits will there be by the end of the year?" Ask for feedback on the biographical information (Pisa, Italian mathematician, 1200s, book called Liber Abaci).

Tell students they are going to solve the rabbit problem.

Give each student a copy of the Fibonacci and the Rabbits Worksheet. Have them work either independently or in groups to come up with an answer. (The answer is 144 pairs of rabbits.)

Ask students if any of the numbers they came up with during the rabbit problem look familiar. Refer back to their count of natural objects and the numbers in the Fibonacci sequence. Explain to the students that the set of numbers that Fibonacci came up with as he solved the rabbit problem appears in all sorts of ways in nature.

Provide your students with a **Focus for Media Interaction** by asking them to check their work along with the narrator in this next section of the video. You may need to remind them that each block represents a pair of rabbits, not a single rabbit.

Play the video clip from where you left off with the narrator's question. It shows students solving the rabbit problem with blocks. The numbers should look familiar to your students after their work with the Fibonacci and the Rabbits Worksheet.

Step 4: Fibonacci and the Natural World
Go to Dr. Ron Knott's Web site at
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html - Rabbits.

Rabbits: Show students the rabbit diagram after the "Fibonacci's Rabbits" heading. A solution diagram for 10 months can be found on this Web site. Dr. Knott also brings up the problems with the rabbit problem – no rabbits die; each pair has exactly one pair of babies every single month starting at two months old and so on. Discuss these shortcomings with the students but explain that the numbers still appear in nature.

Spiral Shells: Scroll down about a third of the way through the Web site (past the honey bees, cows and so on) to "Fibonacci Rectangles and Shell Spirals." There is an animated drawing that shows how a spiral shape can develop by using a series of squares of progressively larger sizes. The size of a square's side is determined by the sum of the sides of the last two squares. The animated drawing is on the left and on the right is a still drawing. Allow it to play while you point out what is happening as the spiral grows, referencing the still drawing as needed.

Once students seem to have a grasp of the creation of the spiral, you can begin to show them how the spiral appears in nature. The first stop is the cross section of a chambered Nautilus shell on that same section of the site. If you have one of these shells to bring in and share, so much the better!

Plants: Continuing to scroll down, there are examples of branching plants, petals on flowers and seed heads. Depending on how much time you have and the level of detail you want to go into, there are links that can be followed for more pictures that illustrate the appearance of Fibonacci numbers.

Pinecones: The next section helps students see how the scales of pinecones are arranged in Fibonacci numbers of spirals. You can click on buttons to see the two lines of spiral scales highlighted. Pass around real pinecones (from the sets of natural materials in the Introductory Activity) so students can compare them with what is on the screen.

Branches: The next section is about how the branches of plants are arranged in a spiral format so that each leaf gets the maximum amount of sunlight. The picture based on the African violet that you can get to by clicking on the "picture" word link is a very good illustration of that. The "leaves per turn" section here relates to ratios of Fibonacci numbers, which you can use as an extension of this basic lesson. Show a real African violet if you have one.

Vegetables: Finally, there is a section on spirals in vegetables, illustrated by the cauliflower. Clicking the buttons will highlight the two spirals on the cauliflower. Again, a real cauliflower can also be examined by students, as can a pineapple.

Culminating Activity

Students will apply what they have learned about Fibonacci patterns and numbers to the human body.

Provide a **Focus for Media Interaction** by asking students to decide whether humans have Fibonacci numbers built into them. Scroll to the final section on Dr. Knott's Web site called "Fibonacci Fingers?" and let students think about their hands. Explain the Phi is the Golden Ratio, which can be found by dividing any Fibonacci number by the one right before it. It should come out to be about 1.6, with accuracy getting closer and closer to "The Golden Number" as you divide larger Fibonacci numbers. This is explained by Dr. Knott in the "Fibonacci numbers and the Golden Number" section.

Have students take on Dr. Knott's challenge: "Why not measure your friends' hands and gather some statistics? I'd be interested in your results if you want to email them to me." (fibandphi@ronknott.com)

Cross-Curricular Extensions

Other Web Sites

- This is another of Dr. Ron Knott's sites that provides many links to sites related to Fibonacci numbers.
  http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html
- This is a history of math site that includes a biography of Fibonacci.
  http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Fibonacci.html
- This site has beautiful pictures that are related to Fibonacci spirals by artist Edward S. May.
  http://www.moonstar.com/~nedmay/chromat/fibonaci.htm
- This British site by Brantacan explains Fibonacci numbers and how they are related to flowers, pinecones, pineapples, palm trees, suspension bridges, spider webs, dripping taps, CDs, your savings account and quite a few other things.
  http://www.branta.connectfree.co.uk/fibonacci.htm

- This National Council of Teachers of Mathematics site uses a children's picture book, Bunches and Bunches of Bunnies, to lead into the Fibonacci rabbit sequence. It includes historical information on Fibonacci as well.
  http://www.nctm.org/wlme/wlme6/supp_eight.htm

Science

- Further explore Fibonacci numbers in nature. Challenge students to find other patterns of numbers in crystals and rocks, in the distance of planets from the sun and so on.

Music

- The keys on a piano are set in Fibonacci numbers: 2 black keys plus 3 black keys make 5 black keys; 8 white keys; 13 keys in all. Chords can be related to Fibonacci numbers. Explore making Fibonacci music by playing notes only a Fibonacci number apart.

Art

- Using the Edward S. May artwork on the Web site listed above for inspiration, have students create spiral art that uses the Fibonacci spiral. Instructions for drawing the spirals can be found in Dr. Knott's "Fibonacci and Shell Spirals" section that you taught above.
- Teach the Golden Rectangle to students. A Golden Rectangle has sides with a ratio of Phi. A lot of information can be found on the Web sites listed above.

Architecture

- Have students look for the Golden Rectangle in bridges, buildings and monuments.

Language Arts

- Have students write poetry with a Fibonacci number of lines, such as a haiku.

History

- Research the 1200s. (This was when the Leaning Tower was built in Fibonacci's hometown of Pisa.) Have students try to add, subtract, multiply and divide with Roman numerals as they did before Fibonacci introduced Arabic numbers.

Community Connections

- Walk around the school's neighborhood or a wild area and look for Fibonacci patterns.
- Talk to an artist about how he/she uses the Golden Rectangle, which includes Fibonacci ratios.
- Have a botanist or zoologist come in and talk about Fibonacci numbers and his/her specialty.
- Include a Fibonacci problem as part of a Math Trial around the school or neighborhood.

Name: _____

Fibonacci and the Rabbits

Beginning with a single pair of rabbits, if every month each productive pair bears a new pair, which becomes productive when they are 1 month old, how many rabbits will there be after 12 months? The key to success is to remember that these mythical rabbits are not able to have babies until they are 1 month old and then it takes 1 month of pregnancy before the babies are born. So it is 2 months between the time a pair of rabbits are born and the time they first give birth. The first 3 rows are done for you. Use the chart on the other side of the paper to help you complete this table.

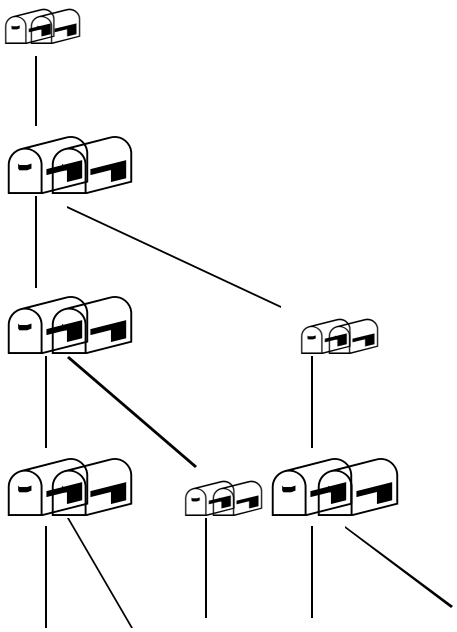
Month	Number of Pairs of Adults	Number of Pairs of Babies	Total Number of Rabbit Pairs
1	0	1	1
2	1	0	1
3	1	1	2
4			
5			
6			
7			
8			
9			
10			
11			
12			

Month 1: One pair of babies, no adults

Month 2: one pair of adults (pregnant)

Month 3: One pair of adults, one pair of babies.

Month 4: first pair, new babies, old babies



Total # of pairs:
1
1
2
3

Month 5:

Month 6:

Month 7:

Month 8:

Month 9:

Month 10:

Month 11:

Month 12: