PYTHAGOREAN THEOREM: LEARNING SQUARED PLUS TEACHING SQUARED EQUALS UNDERSTANDING SQUARED

Subject Matter: Math - Pythagorean Theorem
Grade Levels: 8-12
Time Allotment: Three to five 45-minute class sessions
Master Teacher: Ken McCoy

Overview

This lesson provides students with an in-depth understanding and appreciation for the Pythagorean Theorem through independent exploration. Students will describe, in writing and drawings, a working definition of the Theorem, various proofs of the Theorem and examples of its application. Students will communicate their findings through a team-teaching opportunity and through activities created by the teaching efforts of their peers.

Learning Objectives

Students will be able to:

- Define, communicate and use the Pythagorean Theorem in its various forms.
- Recognize and work with Pythagorean Triples.
- Recognize, work with and reproduce various proofs of the Pythagorean Theorem.
- Define, communicate and use the Pythagorean Theorem in both algebraic and geometric terms.
- Recognize, appreciate and use the Pythagorean Theorem's application to everyday life.

Oregon Standards Available at:

http://www.ode.state.or.us/cifs

Mathematics - Geometry

- Use geometric models and properties of figures (e.g., Pythagorean Theorem) to solve problems.

Media Components

Video

Check the link at http://www.opb.org/edmedia/trs/ to find access to the video(s) from unitedstreaming™ referenced in this lesson plan.

- "Project Mathematics: Theorem of Pythagoras" (22:00)
  - **Clip:** "The Theorem of Pythagoras: Three Questions from Real Life" (02:52)
  - **Clip:** "Discovering the Theorem of Pythagoras" (01:18)
  - **Clip:** "Geometric Interpretation of the Theorem of Pythagoras" (00:27)
  - **Clip:** "Pythagoras" (00:27)
  - **Clip:** "Applying the Pythagorean Theorem" (01:15)
  - **Clip:** "Pythagorean Triples" (00:49)
  - **Clip:** "The Chinese Proof" (01:02)
  - **Clip:** "Euclid's Proof" (01:22)
  - **Clip:** "A Dissection Proof" (00:35)
  - **Clip:** "A Very Simple Proof" (00:40)

Web

- **The Pythagorean Theorem**
  A lesson plan developed by Jacobo Bulaevsky providing hints for students to discover the Pythagorean Theorem on their own. It also includes a review of all the essential material students will need to succeed.
  http://www.arcytech.org/java/pythagoras/

- **Pythagoras' Haven**
  An interactive site illustrating a geometrical proof of Pythagoras' Theorem.
  http://java.sun.com/applets/archive/beta/Pythagoras/

- **Forty-One Pythagorean Proofs**
  An exhaustive list of Pythagorean Theorem proofs developed by Alexander Bogomolny, including those of algebraic and geometric nature.
  http://www.cut-the-knot.org/pythagoras/index.shtml

- **Pythagorean Theorem**
  An interactive approach to Pythagorean Theorem proofs.
  http://www.frontiernet.net/~imaging/pythagorean.html

- **NOVA: Pythagorean Theorem Puzzle**
  From PBS' NOVA program, this site includes a Shockwave illustration of a proof and three examples of how to use the Theorem in real-life applications.
  http://www.pbs.org/wgbh/nova/proof/puzzle/

- **Pythagorean Proof**
  A Web site that animates a unique proof of the Theorem.
  http://www.davis-inc.com/pythagor/index.shtml

- **What Is Your Favorite Pythagorean Proof?**
  A Web Quest lesson for ninth-grade students, created by Crystal L. Furman, complete with a task statement, process details, evaluation rubric and several resources.
  http://www.gowcsd.com/master/ghs/math/furman/pythagor/pythag.htm

Materials

Per Student:

- Several sheets of 1-cm grid paper
- T-square, right triangle and/or protractor
- Ruler
- Pencil
- Scissors

Per Class and/or Group:

- 11" x 17" colored construction paper, enough for at least 2 sheets per student
- Any materials supplied by groups unique to their presentations
- Any unique handouts or worksheets created by groups for use in their presentations
- A computer (or computers) with Internet connectivity, the Windows Media Player installed and downloads of the video clips listed above

Prep for Teachers

Session I: Acquire materials and equipment.

Download and preview the video clips, discovering helpful points to highlight for students during discussion and activities. Acquire the free Window's Media Player to view clips.

Session II: Find examples of typical rubric for suggestions of class-created group evaluation rubric.

Create a list of appropriate Internet sites and search methods to aid students in independent discovery, printing them out as a handout and/or bookmarking them on class computers or creating a Web site with links.

Prepare each computer to be used with the necessary plug-ins for Web sites used (especially the free Flash and Shockwave Players available at http://www.macromedia.com/).

When using media, provide students with a **Focus for Media Interaction**, a specific task to complete and/or information to identify during or after viewing of video segments, Web sites or other multimedia elements.

Introductory Activity

Give the following hypothetical: "My Uncle Bob just emailed me and said he's locked himself out of his house. He left the water running in the bathtub and needs to get back in before it overflows. Walking around the house, he noticed that a window was open on the second floor, 25 feet above the ground. There is also a bush under the window, so he'll have to place the ladder 10 feet from the house. All his neighbors have different length ladders. What length of ladder does he need to borrow?" First, allow students the chance to guess the length of ladder without using math. Next, students can use graph paper to draw a representation of the hypothetical, perhaps seeing if the grid of the graph paper is of help. Students can then use their ruler to devise a scale. Free exploration is important here as students will discover the difficulty of solving the problem without the Pythagorean Theorem. (10 minutes total)

Learning Activities

Session I

Step 1: Present the hypothetical and ask students to use graph paper to present their answers. Have students use rulers to draw and measure their work, reminding them of the need to be neat and organized to aid them in arriving at an accurate answer. (10 minutes)

Step 2: As students work, explain to them that they are working on a problem that is older than your Uncle Bob, maybe older than bathtubs and two-story houses, and maybe even older than ladders.

Step 3: Give basic information on Pythagoras and his approach to the problem. Information gleaned from Web sites and video clips, provided in this lesson plan, can be helpful in this brief informational presentation. (3 minutes)

Step 4: Provide students with a **Focus for Media Interaction** by having them make notes on their drawings from Step 1 while viewing clips from the video, "Project Mathematics: Theorem of Pythagoras" (22:00): 1) "The Theorem of Pythagoras: Three Questions from Real Life" (02:52) and 2) "Discovering the Theorem of Pythagoras" (01:18). **Play** the video clips. (5 minutes)

Step 5: Upon the completion of Step 4, discuss with the class how the algebraic exploration of the Pythagorean Theorem can be difficult to grasp for a beginner. Ask for responses as to why

that might be. Possible answers include: "When dealing with numbers, it can be hard to see what's happening." (5 minutes)

Step 6: Provide students with a **Focus for Media Interaction** by having them think about the discussion in Step 5 while watching the video clips, "Geometric Interpretation of the Theorem of Pythagoras" (00:27) and "Pythagoras" (00:27), from the video, "Project Mathematics: Theorem of Pythagoras" (22:00), and consider how best to solve the difficulty of expressing the Theorem for beginners. **Play** the video clips. (1 minute)

Step 7: State to the class, " 'Rediscovered, again and again' and that's what we're going to do," in an effort to reveal the activities ahead. Explain that they will rediscover the Pythagorean Theorem on their own through independent research and exploration. Explain that your role of teacher will be as support in their efforts, as a resource if needed, as an observer and as a participant in their final activity. (2 minutes)

Step 8: Before answering any questions about what's ahead, provide students with a **Focus for Media Interaction** by asking them, "How would you teach someone this concept of the Pythagorean Theorem?" **Play** the video clip, "Applying the Pythagorean Theorem" (01:15), from the video, "Project Mathematics: Theorem of Pythagoras" (22:00). (2 minutes)

Step 9: Allow for a short discussion regarding some ideas as to how students would teach the concept. Explain to students that for the rest of their study of the Pythagorean Theorem they will be preparing to do just that, teach the Theorem and its various proofs to the class. (10 minutes)

Session II

Step 1: Restate the focus for the group: to teach the Pythagorean Theorem to the class. Explain that in order to do that, one would need to have at least some familiarity and understanding of the Theorem itself. Ask students, "How could you gain that understanding?" Wait for an answer that refers to research, using that statement to explain that students will be involved in research to prepare their lessons. (2 minutes)

Step 2: Students are encouraged to work in teams of two (squared) to explore and illustrate the Pythagorean Theorem and at least two of its proofs, one of which will be taught to the class. Students are grouped, through choice, teacher selection or some other means, and now situate themselves to sit as a group. (3 minutes)

Step 3: Ask students, "Have you ever tried to learn something when the lesson or teacher was boring?" Use their responses to explain that their lessons will need to be well-planned, engaging and involve some activity to explore the concept they're attempting to teach. (2 minutes)

Step 4: Model two lessons, asking students to be ready to discuss positives and negatives of each lesson upon its completion.

Step 1: Explain to students that there are many approaches to the Theorem of Pythagoras and many ways to prove it. The following examples are just two of them.

Step 2: Pass out a sheet of construction paper to each student and have students obtain a pair of scissors, a pencil, a ruler and a t-square, right triangle or protractor. Ask students to position their paper horizontally on their desk. In the center of the paper, have students draw a right triangle with a vertical side of 2" in length and a horizontal side of 4" in length. (3 minutes)

Step 3: Provide students with a **Focus for Media Interaction** by asking them to follow along with your instruction and the video clip, "A Dissection Proof" (00:35), from the video, "Project Mathematics: Theorem of Pythagoras" (22:00). **Play** the video clip. (1 minute)

Step 4: Pause the video clip when the voiceover says, "… Here's one you can do with a pair of scissors." Check to be sure all students have created a correctly positioned right triangle as per your specifications. (1 minute)

Step 5: Have students shade in the right triangle they created.

Step 6: Next, students should create squares using the three sides of the right triangle: one 2" x 2" square using the 2" vertical side of the triangle, one 4" x 4" triangle using the 4" side of the triangle and one square using the hypotenuse of the right triangle. (3 minutes)

Step 7: When students are ready, **play** the video clip from where you left off. **Pause** the video clip again when the voiceover says, " … and trace around it in the middle of the large square." Have students find the middle of their large, hypotenuse-created squares by drawing diagonals from each corner and drawing their new 2" x 2" squares using that mark. Remind students to be as exact and careful as they can be. After drawing their new squares, they may want to have a classmate check their work. (2 minutes)

Step 8: When students are ready, **play** the video clip from where you left off. **Pause** the video clip at the point the voiceover says, " … make cuts along the sides and out to the edges." Have students draw these cut lines before they proceed with cutting to ensure accuracy. When they are ready, they may cut out the large square and follow their drawing for the additional cuts. Students may then reassemble their pieces in their original placement and wait for further instructions. (3 minutes)

Step 9: When students are ready, **play** the video clip from where you left off. **Stop** the video clip when the animation is complete and the screen fades to black. Allow students exploration time. (1 minute)

Step 10: Upon their completion, ask students to discuss the strengths, weaknesses and effectiveness of this activity in their work groups. (1 minute)

Step 11: While students discuss in their groups, prepare a T-table on the board entitled "Paper Cutting Activity" with one side marked "Positives" and the other marked "Negatives."

Step 12: Invite comments to the class from each work group, marking comments in the appropriate sides of the T-table. Possible comments may include: "Hands on activities make it easier and more fun …" "With all the steps, you have to really plan well …" "It was easy to stay on task …" "It was easy to get off task …" etc. (3 minutes)

Step 13: Provide students with a **Focus for Media Interaction**, asking them to discuss in their groups the strengths, weaknesses and effectiveness of the interactive activity on the following Web page. With no instructions, take students to the NOVA Pythagorean Theorem Puzzle Web site (http://www.pbs.org/wgbh/nova/proof/puzzle/). (4 minutes)

Step 14: While students discuss in their groups, prepare a T-table on the board entitled "Puzzle Web Site" with one side marked "Positives" and the other marked "Negatives."

Step 15: Ask student groups to share points from their discussion regarding the Web site, marking comments in the appropriate areas of the new T-table. Possible responses may be similar to those in Step 12, possibly favoring the negative side. (3 minutes)

Step 16: In their work groups, have students discuss ways to strengthen items in the "weakness" side of each T-table. (3 minutes)

Step 5: As students discuss in their groups, teacher passes out copies of an example assessment rubric as a guide for the next step.

Step 6: Guided by the teacher, the class will design a rubric to be used in evaluation of teaching teams. Once created, a student or group will be responsible for creating the form and disseminating it to the class, one copy for each group per student. (6 minutes)

Step 7: Teacher passes out the list of acceptable Web sites and search engines for use in the project.

Step 8: Students will use the suggested Internet sites and search terms to independently extend their learning about the Theorem and the various proofs.

Step 9: Students will have one to three workdays for exploration and preparation of their presentations, during which time groups are free to collaborate with other groups to share information and teach concepts. Early in the workday schedule, it may be helpful to publish for

the class's benefit which work groups plan to cover which proofs, to help lower the chances of a proof being taught multiple times.

Culminating Activity

Students will team teach their findings and illustrations of their assigned proofs. Presentations will include lesson presentation, activity and assessment, as well as a chance for the class to evaluate the team via the class-designed rubric. Depending on class size, number of groups and time planned by each group, this activity will undoubtedly take multiple sessions to complete.

Cross-Curricular Extensions

Health & Physical Education

- When studying the Pythagorean Theorem, students can determine the distances from home plate to the pitcher's mound and second base in baseball and softball.

Mathematics

- Students can create complex two-dimensional figures (or three-dimensional figures) and use the Pythagorean Theorem to find the lengths of the sides for pentagons, hexagons, tetrahedrons, etc.

History and Social Studies

- Students can write and present a brief talk on Pythagoras, his life and their experiences with the lab.

Community Connections

- Invite an architect and/or a building contractor to speak to the class on their use of the Pythagorean Theorem.